SYSTEMATIC APPLICATION OF DUNCAN'S MULTIPLE RANGE TEST TO BIOLOGICAL RESEARCH DATA

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INTRODUCTION

After completing analysis of variance, Appendix Form 1, a researcher has several tests available for evaluating the significance of differences between means (*2, 6*).[2] These tests require the researcher to scrutinize the data and declare the significance of differences. Among these tests, Duncan's (*2*) holds a middle position in the reported number of significant differences. For comparing two means only, the Least Significant Difference (LSD) (*3, 8*) test is identical with Duncan's and most other multiple range tests (table 1 from *2* and *6*). Also for some special comparisons, the LSD test is adequate and preferred by some researchers. However, as the number of means in a range or array increases above two, larger differences than LSD may occur by chance. The Newman-Keuls test (*5, 7*) corrects for this probability. For some research data, however, the Newman-Keuls test is too conservative. Duncan's test values (table I) are intermediate between the LSD values that allow reporting of too many "significant" differences (type I mistake) and Newman-Keuls values that allow the reporting of too few significant differences (type II mistake).

TABLE 1.–*Comparisons of significant values at the 5-percent level of significance for various numbers of means, assuming a variance of unity and infinite degrees of freedom for experimental error*[1]

Type of test procedure	Differences needed for significance when number of means in group are–					
	2	4	8	10	20	100
Least Significant Difference (LSD) (*3, 8*)	2.77	2.77	2.77	2.77	2.77	2.77
Duncan's Multiple Range (*2*)	2.77	3.02	3.23	3.29	3.47	3.67
Newman-Keuls (*5, 7*)	2.77	3.63	4.29	4.47	5.01	5.93

[1] Compare at ∞ in table 2 for 5 percent level of significance. The 3 tests relate similarly for the 1 percent level of significance (*3, 5, 7, 8*, and table 3).

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[2] Italic numbers in parentheses refer to Literature Cited, page 10.

TABLE 2.—*Significant Studentized range r_p values for the 5-percent level of significance*[1]

Partial array / Error df $(n_2)^2$ (p)	Subset size (number of means in that part of the array under immediate scrutiny)															
	2	3	4	5	6	7	8	9	10	12	14	16	18	20	50	100
1	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97
2	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085
3	4.501	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516
4	3.927	4.013	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033
5	3.635	3.749	3.797	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814
6	3.461	3.587	3.649	3.680	3.694	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697
7	3.344	3.477	3.548	3.588	3.611	3.622	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626
8	3.261	3.399	3.475	3.521	3.549	3.566	3.575	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579
9	3.199	3.339	3.420	3.470	3.502	3.523	3.536	3.544	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547
10	3.151	3.293	3.376	3.430	3.465	3.489	3.505	3.516	3.522	3.526	3.526	3.526	3.526	3.526	3.526	3.526
11	3.113	3.256	3.342	3.397	3.435	3.462	3.480	3.493	3.501	3.509	3.510	3.510	3.510	3.510	3.510	3.510
12	3.082	3.225	3.313	3.370	3.410	3.439	3.459	3.474	3.484	3.496	3.499	3.499	3.499	3.499	3.499	3.499
13	3.055	3.200	3.289	3.348	3.389	3.419	3.442	3.458	3.470	3.484	3.490	3.490	3.490	3.490	3.490	3.490
14	3.033	3.178	3.268	3.329	3.372	3.403	3.426	3.444	3.457	3.474	3.482	3.484	3.485	3.485	3.485	3.485
15	3.014	3.160	3.250	3.312	3.356	3.389	3.413	3.432	3.446	3.465	3.476	3.481	3.481	3.481	3.481	3.481
16	2.998	3.144	3.235	3.298	3.343	3.376	3.402	3.422	3.437	3.458	3.470	3.477	3.478	3.478	3.478	3.478
17	2.984	3.130	3.222	3.285	3.331	3.366	3.392	3.412	3.429	3.451	3.465	3.473	3.476	3.476	3.476	3.476
18	2.971	3.118	3.210	3.274	3.321	3.356	3.383	3.405	3.421	3.445	3.460	3.470	3.474	3.474	3.474	3.474
19	2.960	3.107	3.199	3.264	3.311	3.347	3.375	3.397	3.415	3.440	3.456	3.467	3.472	3.474	3.474	3.474
20	2.950	3.097	3.190	3.255	3.303	3.339	3.368	3.391	3.409	3.436	3.453	3.464	3.470	3.473	3.474	3.474
24	2.919	3.066	3.160	3.226	3.276	3.315	3.345	3.370	3.390	3.420	3.441	3.456	3.465	3.471	3.477	3.477
30	2.888	3.035	3.131	3.199	3.250	3.290	3.322	3.349	3.371	3.405	3.430	3.447	3.460	3.470	3.486	3.486
40	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.390	3.418	3.439	3.456	3.469	3.504	3.504
60	2.829	2.976	3.073	3.143	3.198	3.241	3.277	3.307	3.333	3.374	3.406	3.431	3.451	3.467	3.537	3.537
120	2.800	2.947	3.045	3.116	3.172	3.217	3.254	3.287	3.314	3.359	3.394	3.423	3.446	3.466	3.585	3.601
∞	2.772	2.918	3.017	3.089	3.146	3.193	3.232	3.265	3.294	3.343	3.382	3.414	3.442	3.466	3.640	3.735

[1] Values of r_p as amended by Harter (*4*) and abridged from *Biometries* 16(*4*), 1960. Explanatory headings have been added. Multiply table values, for appropriate error degrees of freedom (n_2) and number of means (p) in subset, by constant $s_{\bar{x}}$, to get Duncan's Multiple Range Test values (R_p) at 5-percent level of statistical significance.

[2] Degrees of Freedom for error term used in F-test of means in array.

TABLE 3.—*Significant Studentized range r_p values for the 1-percent level of significance*[1]

Partial array (p) / Error df (n_2)[2]	Subset size (number of means in that part of thearray under immediate scrutiny)															
	2	3	4	5	6	7	8	9	10	12	14	16	18	20	50	100
1	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03
2	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04
3	8.261	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321
4	6.512	6.677	6.740	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756
5	5.702	5.893	5.989	6.040	6.065	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074
6	5.243	5.439	5.549	5.614	5.655	5.680	5.694	5.701	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703
7	4.949	5.145	5.260	5.334	5.383	5.416	5.439	5.454	5.464	5.472	5.472	5.472	5.472	5.472	5.472	5.472
8	4.746	4.939	5.057	5.135	5.189	5.227	5.256	5.276	5.291	5.309	5.316	5.317	5.317	5.317	5.317	5.317
9	4.596	4.787	4.906	4.986	5.043	5.086	5.118	5.142	5.160	5.185	5.199	5.205	5.206	5,206	5,206	5,206
10	4.482	4.671	4.790	4.871	4.931	4.975	5.010	5.037	5.058	5.088	5.106	5.117	5.122	5.124	5.124	5.124
11	4.392	4.579	4.697	4.780	4.841	4.924	4.924	4.952	4.975	5.009	5.031	5.045	5.054	5.059	5.061	5.061
12	4.320	4.504	4.622	4.706	4.767	4.815	4.852	4.883	4.907	4.944	4.969	4.986	4.998	5.006	5.011	5.011
13	4.260	4.442	4.560	4.644	4.706	4.755	4.793	4.824	4.850	4.889	4.917	4.937	4.950	4.960	4.972	4.972
14	4.210	4.391	4.508	4.591	4.654	4.704	4.743	4.775	4.802	4.843	4.872	4.894	4.910	4.921	4.940	4.940
15	4.168	4.347	4.463	4.547	4.610	4.660	4.700	4.733	4.760	4.803	4.834	4.857	4.874	4.887	4.914	4.914
16	4.131	4.309	4.425	4.509	4.572	4.622	4.663	4.696	4.724	4.768	4.800	4.825	4.844	4.858	4.892	4.892
17	4.099	4.275	4.391	4.475	4.539	4.589	4.630	4.664	4.693	4.738	4.771	4.787	4.816	4.832	4.874	4.874
18	4.071	4.246	4.362	4.445	4.509	4.560	4.601	4.635	4.664	4.711	4.745	4.772	4.792	4.080	4.858	4.858
19	4.046	4.220	4.335	4.419	4.483	4.534	4.575	4.610	4.639	4.686	4.722	4.749	4.771	4.788	4.845	4.845
20	4.024	4.197	4.312	4.395	4.459	4.510	4.552	4.587	4.617	4.664	4.701	4.729	4.751	4.769	4.833	4.833
24	3.956	4.126	4.239	4.322	4.386	4.437	4.480	4.516	4.546	4.596	4.634	4.665	4.690	4.710	4.802	4.802
30	3.889	4.056	4.168	4.250	4.314	4.366	4.409	4.445	4.477	4.528	4.569	4.601	4.628	4.650	4.772	4.777
40	3.825	3.988	4.098	4.180	4.244	4.296	4.339	4.376	4.408	4.461	4.503	4.537	4.566	4.591	4.740	4.764
60	3.762	3.922	4.031	4.111	4.174	4.226	4.270	4.307	4.340	4.394	4.438	4.474	4.504	4.530	4.707	4.765
120	3.702	3.858	3.965	4.044	4.107	4.158	4.202	4.239	4.272	4.327	4.372	4.410	4.442	4.469	4.673	4.770
∞	3.643	3.796	3.900	3.978	4.040	4.091	4.135	4.172	4.205	4.261	4.307	4.345	4.379	4.408	4.635	4.776

[1] Values of r_p as amended by Harter (*4*) and abridged from *Biometrics* 16(*4*), 1960. Explanatory headings have been added. Multiply table values, for appropriate error degrees of freedom (n_2) and number of means (p) in subset, by constant $s_{\bar{x}}$, to get Duncan's Multiple Range Test values (R_p) at 1-percent level of statistical significance.

[2] Degrees of Freedom for error term used in F-test of means in array.

DATA, DETAILS, FORMULAS, AND TABLES NEEDED FOR DUNCAN'S TEST

A. *Means for testing (table 4).*

Table 4 gives three sets of means adapted from a horticultural crops research project (*9*). Usually, three or more means are tested. If a set contains only two, the F-test in the analysis of variance already indicates the degree of separation between the two. However, Duncan's test can be applied as an added check.

B. *Details from analysis of variance of data (table 5).*

1. Error variance or s^2.
2. Degrees of freedom (n_2) for error variance.
3. Number (n) of items averaged to obtain each mean.

When means come from interaction data from two or more main effects having one error term in common, use that common error term (s^2) and its degrees of freedom (n_2) for error, as the appropriate s^2 for Duncan's test. When interaction data involve two or more error terms with separate degrees of freedom, refer to Koch[3] and Cochran and Cox (*1* pp. 297-305). And, if possible, consult a biometrician or statistician. These sources can help compute a weighted error term and degrees of freedom for use much as s^2 and n_2 for the single simple error term are used. Number (n) of items averaged into a mean is sometimes the number of true replicates but is often some other number.

TABLE 4.—*Percentage of decay in samples of inoculated punctured apples following cold (21° C.) of hot (45° C.) chemical-dip treatment and a short storage*[1]

Type of treatment	Chemical treatment code number								Average
	1	2	3	4	5	6	7	8	
	Pct.	*Pct.*	*Pct.*	*Pct.*	*Pct.*	*Pct.*	*Pct.*	*Pct.*	*Pct.*
Cold dip	85.000	8.333	9.333	12.667	13.333	3.333	10.000	15.333	19.667
Hot dip	89.000	5.333	9.000	9.333	13.000	1.000	2.333	13.667	17.833
Average	87.000	6.833	9.167	11.000	13.170	2.167	6.167	14.500	18.750

[1] Original data from a horticultural crops research project are based on 48 treated, 100-fruit samples.

TABLE 5.—*Analysis of variance of percentage of decay (blue mold) in 48 treated, 100-fruit samples of inoculated punctured apples*

Source of variation	Number of squares	Items per square (n)	Degrees of freedom (n_2)	Variance (s^2)	F-test significance[1]
Total	48	1	47	728.02	—
Replicates	3	16	2	210.50	**
Dip chemicals	8	6	7	4658.14	**
Dip temperatures	2	24	1	40.00	NS
Chemical x temperature	16	3	7	16.43	NS
Error	—	—	30	34.46	—

[1] ** = Significant at the 1-percent level of statistical significance. NS = not significant.

Error variance (s^2) is sometimes called error mean square, or residual experimental error. Note that s^2 is already squared. Degrees of freedom, which we often refer to as "df," is called n_2 by Duncan. Because n_2 is easily confused with n, note carefully whether the formula used requires n or n_2. When means for testing have one error term as do means that come from one main effect, Duncan's test requires the use of the error term (s^2) and its degrees of freedom (n_2) that was previously used to test for the significance of differences among those means.

Also, Duncan's test must be applied to interaction data before letters are appended. A nonsignificant interaction does not imply that individual means in the interaction data are not significantly different from each other.

[3] Koch, E.J. Presentation of Experimental Results, 12 pp. Paper presented at symposium of Amer. Soc. Hort. Sci. and ENAR Biometric Soc., Pa. State Univ., University Park, Pa., Aug. 31, 1959.

C. Formula for standard error of the mean

$$s_{\bar{x}} = \sqrt{\frac{s^2}{n}}$$

The partly verbalized formula reads, "standard error of the mean = $\sqrt{\frac{\text{error variance}}{\text{number of items in mean}}}$". To solve, first get numerical values for s^2 and n from the analysis of variance and then substitute these values in the formula. Use $s_{\bar{x}}$ as a constant to generate Duncan's significant range (R_p) values in section E.

D. Tables of significant Studentized ranges (r_p) (tables 2 and 3).

Tables 2 and 3, significant Studentized ranges (r_p) for 5- and 1-percent significance levels, were abridged from Harter (*4*). Tables for significance levels of 10, 0.5, and 0.1 percent are also available. The values in the first column of Studentized ranges are identical with values used for computing LSD values (see Introduction and table 1).

E. Formula for Duncan's significant range value ($R_p = s_{\bar{x}} \cdot r_p$).

To get the Duncan value R_p, multiply the standard error of the mean ($s_{\bar{x}}$) from the formula in section C by the r_p value from Studentized range in table 2 or 3.

R_p = Duncan's significant range value for p means.

$s_{\bar{x}}$ = standard error of the mean.

r_p = Studentized range value from the 5- or 1-percent table.

In the r_p table, the n_2 stands for the number of degrees of freedom in the error term. The p stands for the number of means in the partial array or subset. To get the r_p value, enter the 5- or 1-percent table at the correct degrees of freedom for error (n_2) and go across to the correct number of means (p) in the subset of the array under scrutiny.

PROCEDURE FOR DUNCAN'S TEST AND LABELING OF MEANS

A. Compute and record the analysis of variance (table 5).

Table 5 summarizes the analysis of data on the percentage of decay after storage in 48 treated samples, each containing 100 inoculated punctured apples. The experiment comprised three replicates of 16 dip treatments consisting of eight chemicals applied at two temperatures, 21° and 45° C. Means for the treatment main effects (dip-chemical, and dip-temperature) and for the interaction of chemical x temperature appear in table 4. Means were computed to three decimal places for Duncan's test. After testing, these figures can be rounded to practical values for publication.

B. Compute constant ($s_{\bar{x}} = \sqrt{\frac{s^2}{n}}$) for each set of treatment means (table 6).

Each of the three treatment items has the same s^2 but a different number (n) of items included in each mean.

1. Dip chemicals:

$$s_{\bar{x}} = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{34.46}{6}} = \sqrt{5.7433} = \underline{\underline{2.3965}}$$

2. Dip temperatures:

$$s_{\bar{x}} = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{34.46}{24}} = \sqrt{1.4358} = \underline{\underline{1.1981}}$$

3. Interaction (chemical x temperature):

$$s_{\bar{x}} = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{34.46}{3}} = \sqrt{11.4867} = \underline{\underline{3.3892}}$$

C. Compute and tabulate Duncan's significant range (R_p) values (table 6).

Enter 5- and 1-percent tables of significant Studentized range (r_p) values at 30 degrees of freedom for error (n_2) to get values for comparing means within subsets of from two to 14 means. Multiply these values by the constant $s_{\bar{x}}$ values as computed above to get Duncan's significant range (R_p) values (table 6). Use these Duncan's (R_p) values in separating significantly different means from each other and in grouping into ranges the means that are not significantly different from each other.

D. Arrange means into descending or ascending arrays (table 7).

The means in each array can be arranged from highest to lowest, or from lowest to highest, in either a vertical column or in a horizontal row.

E. Determine upper and lower limits of each range in each array (table 7).

Assign Duncan's letters as each test proceeds. Letter assignment follows the numerical order shown by superscript numbers above Duncan's letters in table 7. We start with a very simple example below to intro-

TABLE 6.—*Computed Duncan's significant range (R_p) values for Duncan's Multiple Range Test of main effect and interaction means for dip temperature and dip chemical*[1]

Subset size — Number of means	5-percent significance level: r_p at 30 degrees of freedom	5-percent: Dip chemical $s_{\bar{x}}$ (constant) 2.3965	5-percent: Dip temperature; $s_{\bar{x}}$ (constant) 1.1981	5-percent: Interaction chem. x temp. $s_{\bar{x}}$ (constant) 3.3892	1-percent significance level: r_p at 30 degrees of freedom	1-percent: Dip chemical $s_{\bar{x}}$ (constant) 2.3965	1-percent: Dip temperature; $s_{\bar{x}}$ (constant) 1.1981	1-percent: Interaction chem. x temp. $s_{\bar{x}}$ (constant) 3.3892
		R_p	R_p	R_p		R_p	R_p	R_p
2	2.888	6.921	3.460	9.817	3.889	9.320	4.659	13.181
3	3.035	7.273	—	10.317	4.056	9.720	—	13.747
4	3.131	7.503	—	10.642	4.168	9.989	—	14.126
5	3.199	7.666	—	10.874	4.250	10.185	—	14.404
6	3.250	7.789	—	11.047	4.314	10.338	—	14.621
7	3.290	7.884	—	11.183	4.366	10.463	—	14.797
8	3.322	7.961	—	11.292	4.409	10.566	—	14.943
9	3.328	—	—	11.312	4.445	—	—	15.065
10	3.352	—	—	11.394	4.477	—	—	15.173
11	3.389	—	—	11.485	4.504	—	—	15.265
12	3.405	—	—	11.540	4.528	—	—	15.346
13	3.418	—	—	11.584	4.550	—	—	15.421
14	3.430	—	—	11.625	4.569	—	—	15.485

[1] Multiply $s_{\bar{x}}$ value as a constant by each of the appropriate r_p values to get R_p values.

TABLE 7.—*Classification by Duncan's Multiple Range Test of 3 series of means arrayed in descending order*[1]

Treatment and code[2]	Mean[1]	Mean separation at 5-percent significance level[2]: Duncan's test letters[3]	Mean separation at 5-percent significance level[2]: Duncan's test lines	Mean separation at 1-percent significance level[2]: Duncan's test letters[3]	Mean separation at 1-percent significance level[2]: Duncan's test lines
	Percent				
Dip temperatures:					
C (Cold 21° C.)	19.667	a^1		a^1	
H (Hot 45° C.)	17.833	a^2		a^2	
Dip chemicals:					
1	87.000	a^1		a^1	
8	14.500	b^2		b^2	
5	13.167	b^3 c^9		b^3	
4	11.000	b^4 c^8		b^4 c^{12}	
3	9.167	b^5 c^7 d^{14}		b^5 c^{11}	
2	6.833	c^6 d^{13}		b^6 c^{10}	
7	6.167	$c^{10}$$d^{12}$		b^7 c^9	
6	2.167	d^{11}		c^8	
Interaction (chem. x temp.)					
1H	89.000	a^1		a^1	
1C	85.000	a^2		a^2	
8C	15.333	b^3		b^3	
8H	13.667	b^4 c^{24}		b^4	
5C	13.333	b^5 c^{23}		b^5	
5H	13.000	b^6 c^{22}		b^6	
4C	12.667	b^7 c^{21}		b^7	
7C	10.000	b^8 $c^{20}$$d^{34}$		b^8	
4H	9.333	b^9 $c^{19}$$d^{33}$		b^9	
3C	9.333	$b^{10}$$c^{18}$$d^{32}$		b^{10}	
3H	9.000	$b^{11}$$c^{17}$$d^{31}$		b^{11}	
2C	8.333	$b^{12}$$c^{16}$$d^{30}$		b^{12}	
2H	5.333	$b^{13}$$c^{15}$$d^{29}$		b^{13}	
6C	3.333	$c^{14}$$d^{28}$		b^{14}	
7H	2.333	$c^{25}$$d^{27}$		b^{15}	
6H	1.000	d^{26}		b^{16}	

[1] Means of decay percentage in inoculated punctured apples after a short time of cold storage following 8 chemical dip treatments. Data based on 48 treated, 100-fruit apple samples. [2] Data were processed by the analysis of variance and Duncan's Multiple Range Test. Comparable means followed by no letters (or lines) in common are significantly different at the 5-percent (or 1-percent) level of statistical significance. [3] Duncan's letters after means were placed in order noted by the superscript number over each letter. This order is also outlined in the text and is shown here for illustration.

duce the method. With practice, such a test can be done by inspection. Later examples are somewhat more involved but also easy to follow.

1. Testing at the 5-percent level of statistical significance:

a. Temperature treatment means (5-percent significance level)–

(1) Assign Duncan's letter "a" to 19.667, which is the upper limit of the range under view (table 7).

(2) From 19.667 subtract 3.460, Duncan's value (R_p 5 percent) for 2 means (from table 6), to get 16.207.

$$\begin{array}{r} 19.667 \\ -3.460 \\ \hline 16.207 \end{array}$$

Decay percentage 16.207 is lower than the next mean, 17.833. Therefore, 17.833 is in range limits of 19.667. Assign Duncan's letter a to 17.833. Since 17.833 is also the last number in the series, it completes the range and also the Duncan's test for the series. In the range in which 19.667 a is the highest number, 17.833 a is the lowest.

(3) Another check can be applied to verify these conclusions. From 19.667 subtract 17.833 to get 1.834:

$$\begin{array}{r} 19.667 \\ -17.833 \\ \hline 1.834 \end{array}$$

The difference, 1.834, is less than the difference, 3.460, required for significance by Duncan's significant range (R_p) value. Therefore, 19.667 and 17.833 are not significantly different and both require the same Duncan's letter a postscript. Thus, we have completed this section of the testing at the 5-percent level.

b. Chemical treatment means (5-percent significance level)–

(1) Assign Duncan's letter a to 87.000, which is the upper limit in the range under view (table 7).

(2) From 87.000 subtract 6.921, Duncan's value (R_p5-percent) for two means (from table 6), to get 80.079:

$$\begin{array}{r} 87.000 \\ -6.921 \\ \hline 80.079 \end{array}$$

Decay percentage 80.079 is higher than 14.500; therefore, 14.500 is outside the range limits containing 87.000 a. To show this separation, we assign Duncan's letter "b" to 14.500, placing the b one letter space to the right of the a column to avoid confusion. Inasmuch as there is no intermediate value in the series between 87.000 a and 14.500 b, each belongs to a separate range. Uniquely in this series, 87.000 is in a range with only one item which is thus both the upper and lower limit.

As a check add 6.921 to 14.500 to get 21.421.

$$\begin{array}{r} 14.500 \\ +6.921 \\ \hline 21.421 \end{array}$$

Decay percentage 21.421 is less than 87.000. Therefore 14.500 is outside the range of 87.000 and also is the top of a new range.

(3) Now, resume testing with 14.500 as the upper limit of the second range under view. From 14.500 subtract 6.921, Duncan's R_p value for 2 means, to get 7.579:

$$\begin{array}{r} 14.500 \\ -6.921 \\ \hline 7.579 \end{array}$$

Decay percentage 7.579 is below 9.167. Therefore, 13.167, 11.000, and 9.167 are in the range with 14.500 and accordingly receive Duncan's letter b.

Continue down, by testing 6.833. Count 14.500 as the first mean in the range. The second mean, then, is 13.167, and the fifth mean is 6.833. Thus, from 14.500 subtract Duncan's R_p value for 5 means, 7.666, to get 6.834.

$$\begin{array}{r} 14.500 \\ -7.666 \\ \hline 6.834 \end{array}$$

Decay percentage 6.834 is above 6.833. Therefore, 6.833 is outside the range of 14.500 and receives Duncan's letter "c." To avoid confusion, place the c in the column, one letter space to the right of b.

(4) We have found that 6.833 c is in a range separate from 14.500 b. We now proceed to find the upper and then the lower limit of the new range that contains 6.833. To 6.833 add Duncan's R_p value for two means, 6.921, to get 13.754.

$$\begin{array}{r} 6.833 \\ +6.921 \\ \hline 13.754 \end{array}$$

Decay percentage 13.754 is above 13.167. Therefore, all values above 6.833, up to and including 13.167, are in the range of 6.833. Thus, assign letter c to values 9.167, 11.000, and 13.167 in addition to the letter b already assigned to these values. We have already shown that 14.500 is in a range separate from 6.833; thus, 13.167 is the upper limit of the range containing 6.833. Now, test to find the lower limit of that range.

From 13.167, as first mean of the range, count 11.000 as the second mean. Pass 6.833, which we have already tested continue to 6.167, as the fifth mean.

Next, from 13.167 subtract Duncan's R_p value for five means, 7.666, to get 5.501:

$$\begin{array}{r} 13.167 \\ -7.666 \\ \hline 5.501 \end{array}$$

Decay percentage 5.501 is below 6.167. Therefore, 6.167 is in the range of 13.167 and receives Duncan's letter c. Now, test for inclusion (or exclusion) of 2.167.

From 13.167 subtract, Duncan's R_p value for six means, 7.789, to get 5.378.

$$\begin{array}{r} 13.167 \\ -7.789 \\ \hline 5.378 \end{array}$$

Decay percentage 5.378 is above 2.167. Therefore, 2.167 is outside the range and receives Duncan's letter "d." Put d one letter space to the right of the c column to avoid confusion. As 2.167 d is shown to be outside the range of 13.167 "bc," 6.167 is shown to be the lower limit of the range with 13.167 as the upper limit.

(5) Because 2.167 d is the last value in the series, it is also the lower limit of the last range in the series. We now proceed to find the upper limit of the range that includes 2.167.

To 2.167 add Duncan's R_p value for two means, 6.921, to get 9.088:

$$\begin{array}{r} 2.167 \\ +6.921 \\ \hline 9.088 \end{array}$$

Decay percentage 9.088 is above 6.833. Therefore, all values above 2.167 d up to and including 6.833 are in the range with 2.167. Thus, we assign Duncan's letter d to 6.167 and 6.833, in addition to letters they already have. Now we test 9.167, the fourth possible mean in the range.

To 2.167 add Duncan's R_p value for four means, 7.503, to get 9.670:

$$\begin{array}{r} 2.167 \\ +7.503 \\ \hline 9.670 \end{array}$$

Decay percentage 9.670 is above 9.167. Therefore, 9.167 is also in the range with 2.167 and is also labeled d.

All that remains to be tested is the value 11.000, the possible fifth mean in the range.

To 2.167 add Duncan's R_p value for five means, 7.666, to get 9.833.

$$\begin{array}{r} 2.167 \\ +7.666 \\ \hline 9.833 \end{array}$$

Decay percent 9.833 is less than 11.000. Therefore, 11.000 is outside the range of 2.167. Also, 9.167 is shown to be the upper limit of the range having 2.167 as its lower limit.

Thus, we have completed (at the 5-percent level) the testing and labeling of the means for dip chemicals.

c. Temperature x chemical interaction (5-percent significance level)–

In separating into ranges the series of means for the temperature x chemical interaction, we use the same method as we used for the preceding two series for the main effects. However, here we describe only the points needed for the reader to review the method which has already been given.

(1) Label 89.000 with Duncan's letter a. Use 9.817 as R_p for two means to show that 85.000 is in the same range and receives Duncan's letter a, and that 15.333 is outside the range and receives Duncan's letter b. Check 15.333 as the upper limit of the new range by adding R_p for two means, 9.817.

(2) Start with 15.333 as the upper limit of the second range and test by successively subtracting R_p values for from two means to 11 and finally 12 means. Thus we determine, in the range starting with 15.333 that 5.333 is the lower limit and that 3.333 is outside the range. Values including and between 15.333 and 5.333 all are given Duncan's letter b label. Mean 3.333 is given Duncan's letter c label.

(3) Now work up from 3.333, finally using R_p for 11 means to determine the top or upper limit of the third range as 13.667. Label all values from 3.333, up to and including 13.667, with Duncan's letter c. Reverse and work down from 13.667, using R_p values for 12 and then 13 means to show that 2.333 is the bottom of the range that includes 13.667, and that 1.000 is outside that range.

(4) Label 1.000 with Duncan's letter d as the bottom value in the last range of the series 89.000 to 1.000. Now work up from 1.000, starting with R_p value for two means and ending with R_p values for nine means, and finally for 10 means. Thus, 10.000 is shown to be the top value or upper limit of the range that has 1.000 as its lower limit, and 12.667 is shown to be outside the range. Values from 1.000 to 10.000 receive Duncan's letter d.

Thus, we have completed the testing (at the 5-percent level) of the means for the temperature x chemical interaction.

2. Testing at the 1-percent level of statistical significance:

With certain exceptions, the testing for mean separation at the 1-percent level of significance is much the same as the testing for the 5-percent level already described. Instead of table 2, use table 3 to find r_p values computed for the 1-percent significance level. Thus, using the same $s_{\bar{x}}$ values but different r_p values, we end with larger R_p values (table 6). Finally, we have fewer significantly different range values for the 1-percent than for the 5-percent level (table 7). For dip-chemicals, 14.500 percent is significantly different from 6.833 percent at the 5-percent level but not at the 1-percent level. Also in the same group of means, four subgroups are found at the 5-percent level but only three at the 1 percent. The researcher has the choice of using 1- or 5-percent significance level.

F. Review of steps in applying Duncan's test to find each upper and lower range limit.

To help the reader to visualize the procedure for applying Duncan's Multiple Range Test to a group of means, the following steps of the procedure are outlined. These steps are also shown diagramatically in figure 1. After analyzing the data, computing $s_{\bar{x}}$ and Duncan's R_p values, arrange the means in descending (or ascending) order and then use computed R_p values to separate the means into subranges.

1. Label top mean of range a.
2. Find and label bottom mean of range a.
3. Move down one mean from bottom of range a to midmean of range b.
4. Work up from midmean b to find and label top mean b.
5. Work down from top mean b to find and label bottom mean b. Steps 6, 7, and 8 and subsequent triads are repeats of steps 3, 4, and 5.
6. Move down one mean from bottom of range b to midmean range c.
7. Work up from midrange c to find and label top mean c.
8. Work down from top mean c to find and label bottom mean c.

9-17. Repeat steps 3, 4, and 5 until the bottom mean of the last range is reached.

18-19. In some series, only one mean remains below the bottom mean of the previous range. In these series, the final step is to work up from the last remaining [botton] mean to find and label the top of its range.

All means between top and bottom means take the same label letter as the top and bottom means. Therefore, some means have more than one letter label.

REPORTING AND INTERPRETING THE MEANS WITH APPENDED DUNCAN'S LETTERS

After applying the Duncan's Multiple Range Test to one or more series of means, the arrays may be arranged back to the original order, as in table 2 or tables 8 and 9. In table 8, Duncan's letters are retained and the percent values rounded. When a table that uses Duncan's Multiple Range Test letters is published, the following footnote may be helpful:

"[1] Data were processed by the analysis of variance and Duncan's Multiple Range Test. Comparable means followed by no letters in common are significantly different at the 5-percent [or 1-percent] level."

When denoting ranges, solid lines as borderlining or underlining can be used rather than letters (tables 7 and 9). When lines are used, the second sentence in the footnote can read:

"[1] . . . .Comparable means with no borderlining or underlining in common are significantly different at the 5-percent [or 1-percent] level."

Significant differences as shown by Duncan's Multiple Range Test letters can be seen in block tables, such as table 8, or in array tables, such as tables 7 and 9. In table 9, means are arranged horizontally for reporting. Many statements can be made from the tabulated data. For example, in tables 7, 8, and 9, examine chemical-dip data tested at the 5-percent level of statistical significance. Each of the other seven chemical dips is significantly more effective than chemical dip 1 in controlling decay. Treatment 6 is also significantly more effective in controlling decay than treatments 1, 4, 5, and 8. No significant difference in decay control is noted among chemical dips 3, 4, 5, and 8; among dips 2, 3, 4, 5, and 7; or among dips 2, 3, 6, and 7.

TABLE 8.–*Percentage of decay in inoculated punctured apples after cold or hot chemical-dip treatments and a short storage*[1]

Chemical-dip code	Amount of decay–[2]		Average
	Cold treatment (21° C.)	Hot treatment (45° C.)	
	Percent	*Percent*	*Percent*
1	85.0a	89.0a	87.0a
2	8.3 bcd	5.3 bcd	6.8 cd
3	9.3 bcd	9.0 bcd	9.2 bcd
4	12.7 bc	9.3 bcd	11.0 bc
5	13.3 bc	13.0 bc	13.2 bc
6	3.3 cd	1.0 d	2.2 d
7	10.0 bcd	2.3 cd	6.2 cd
8	15.3 b	13.7 bc	14.5 b
Average ...	19.7a	17.8a	18.75

[1] Based on 48 dip-treated, 100-fruit samples of apples.
[2] Data were processed by the analysis of variance and Duncan's Multiple Range Test. Comparable means followed by no letters in common are significantly different at the 5-percent level of statistical significance. A table similar to table 8 can be constructed for the 1-percent level of significance from data in table 7. At the 1-percent level, cold and hot treatment values 85.0 and 89.0 and averages 19.7 and 17.8 are labeled a. All other values in the two columns are labeled b. The eight figures for the treatment averages are labeled, respectively, a, bc, bc, bc, b, c, bc, and b. Grand average 18.75 receives no letter label.

TABLE 9.–*Percentage decay in inoculated punctured apples after hot or cold chemical-dip treatments and a short period of cold storage*[1]

Item	Average decay[2] after treatment number–							
	1	8	5	4	3	2	7	6
Means, pct. .	87.0	14.5	13.2	11.0	9.2	6.8	6.2	2.2
Duncan's lines	———							
		———	———	———	———			
			———	———	———	———	———	
					———	———	———	———

[1] Based on 48 dip-treated, 100-fruit samples of apples.
[2] Data were processed by the analysis of variance and Duncan's Multiple Range Test. Means with no underlining in common are significantly different at the 5-percent level of statistical significance.

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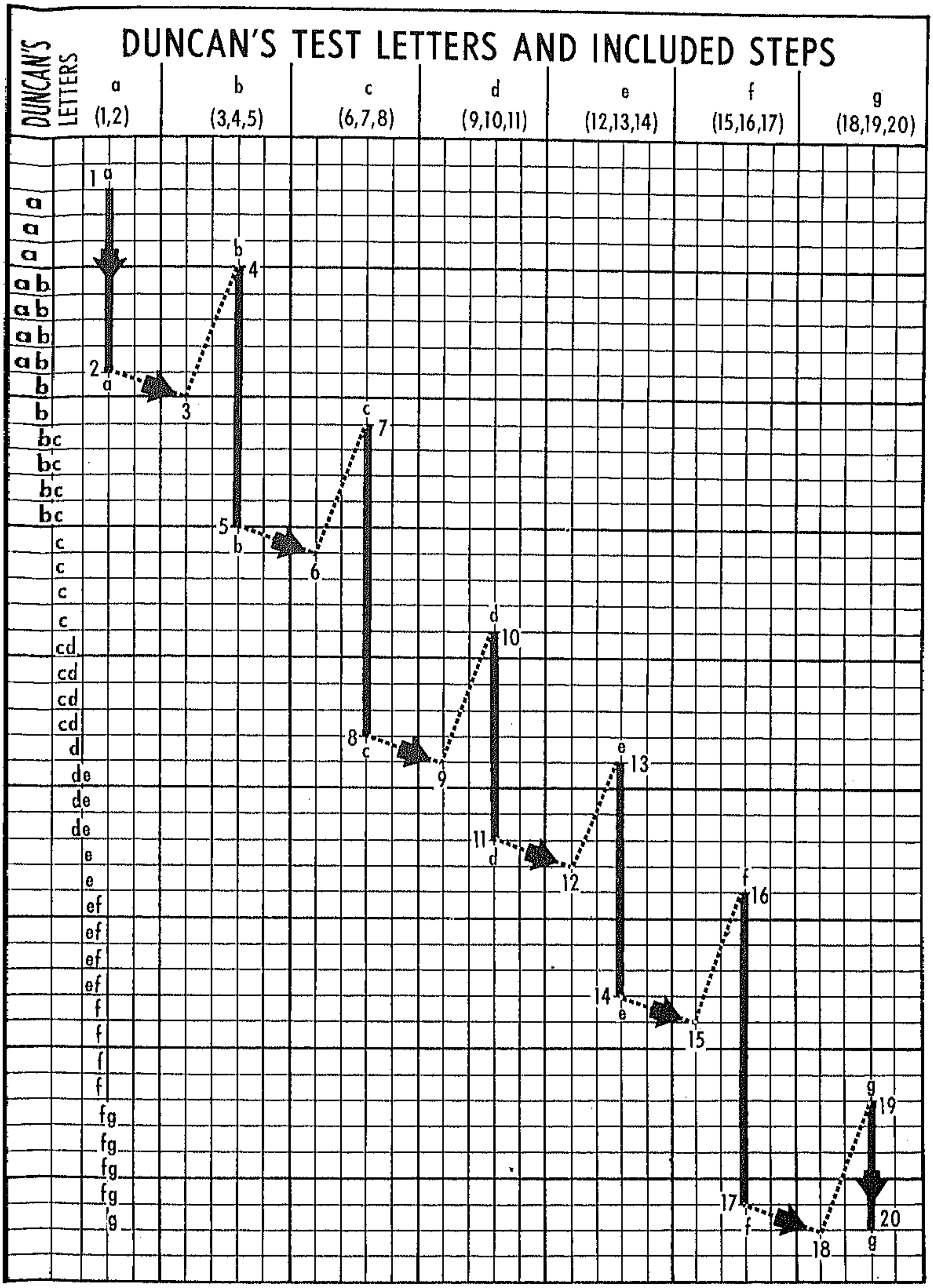

ure 1.–Steps and suggested pathway direction for determining upper and lower limits of each of seven ranges in a hypothetical series of means by Duncan's Multiple Range Test. Letters refer to Duncan's range letters, applied to means from top to bottom in each range. Numbers refer to the sequence of steps to be followed. Heavy vertical lines delineate each range and are analogous to Duncan's test lines in table 7.